

PASCAL'S TRIANGLE

IN THE EARLY 12TH CENTURY, CHINESE MATHEMATICIAN YANGHUI DISCOVERED THE TRIANGLE STUDIED 500 YEARS LATER BY FRENCH MATHEMATICIAN BLAISE PASCAL.

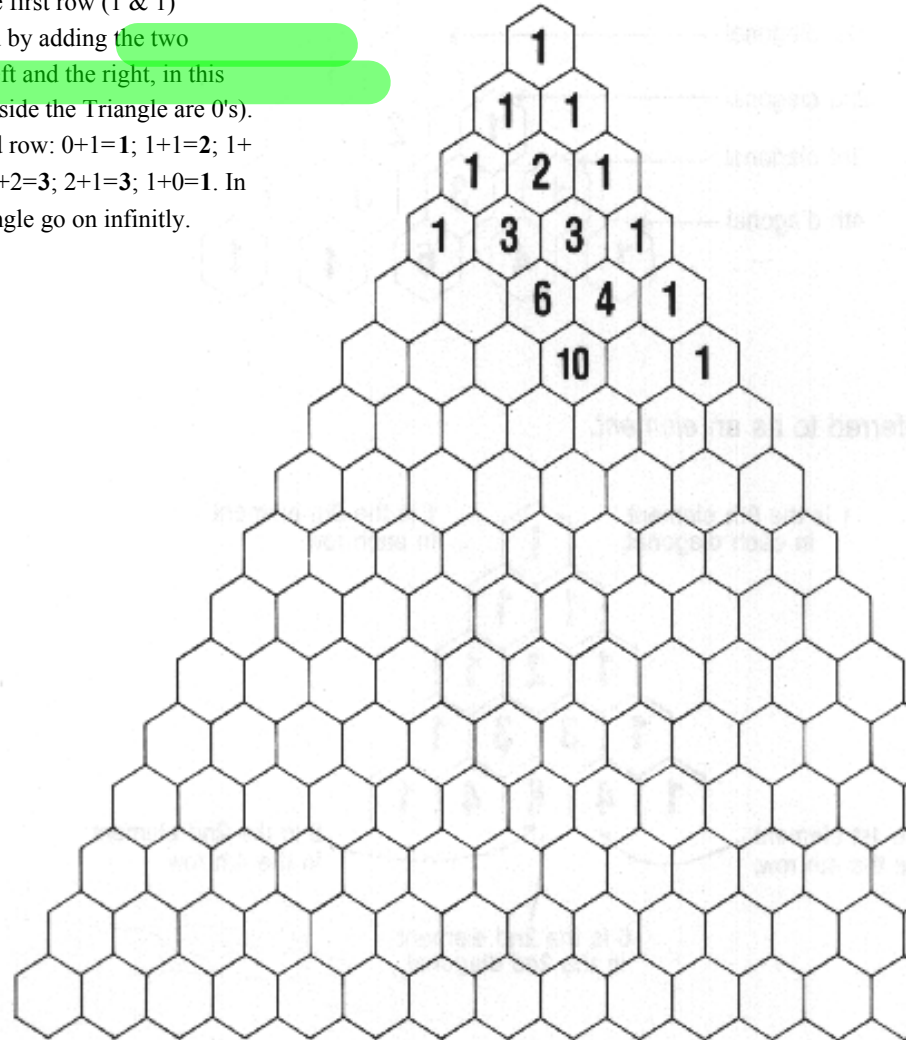
EACH SUBSEQUENT ROW IS OBTAINED BY ADDING THE TWO ENTRIES DIAGONALLY ABOVE

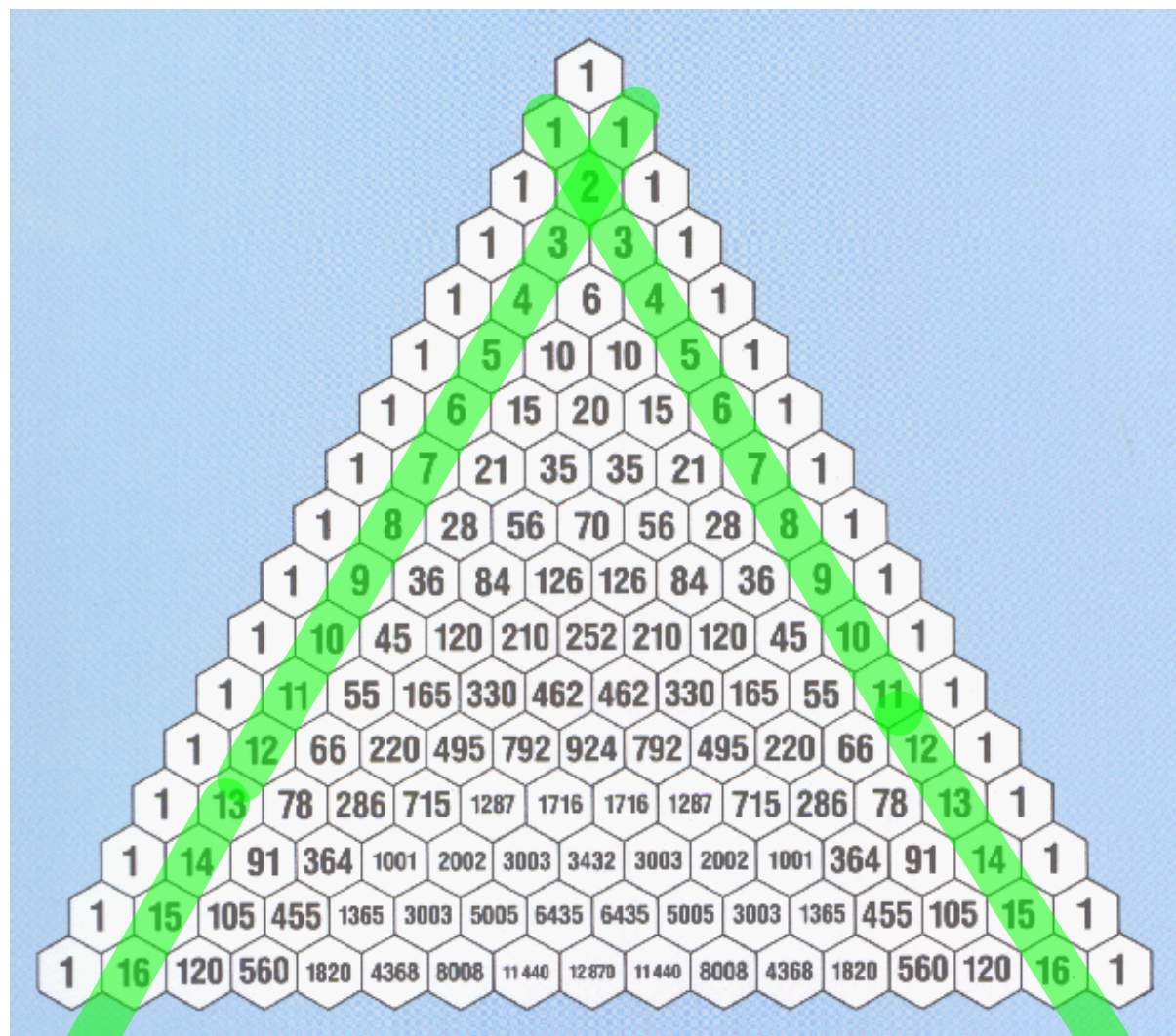


Pascal's Triangle

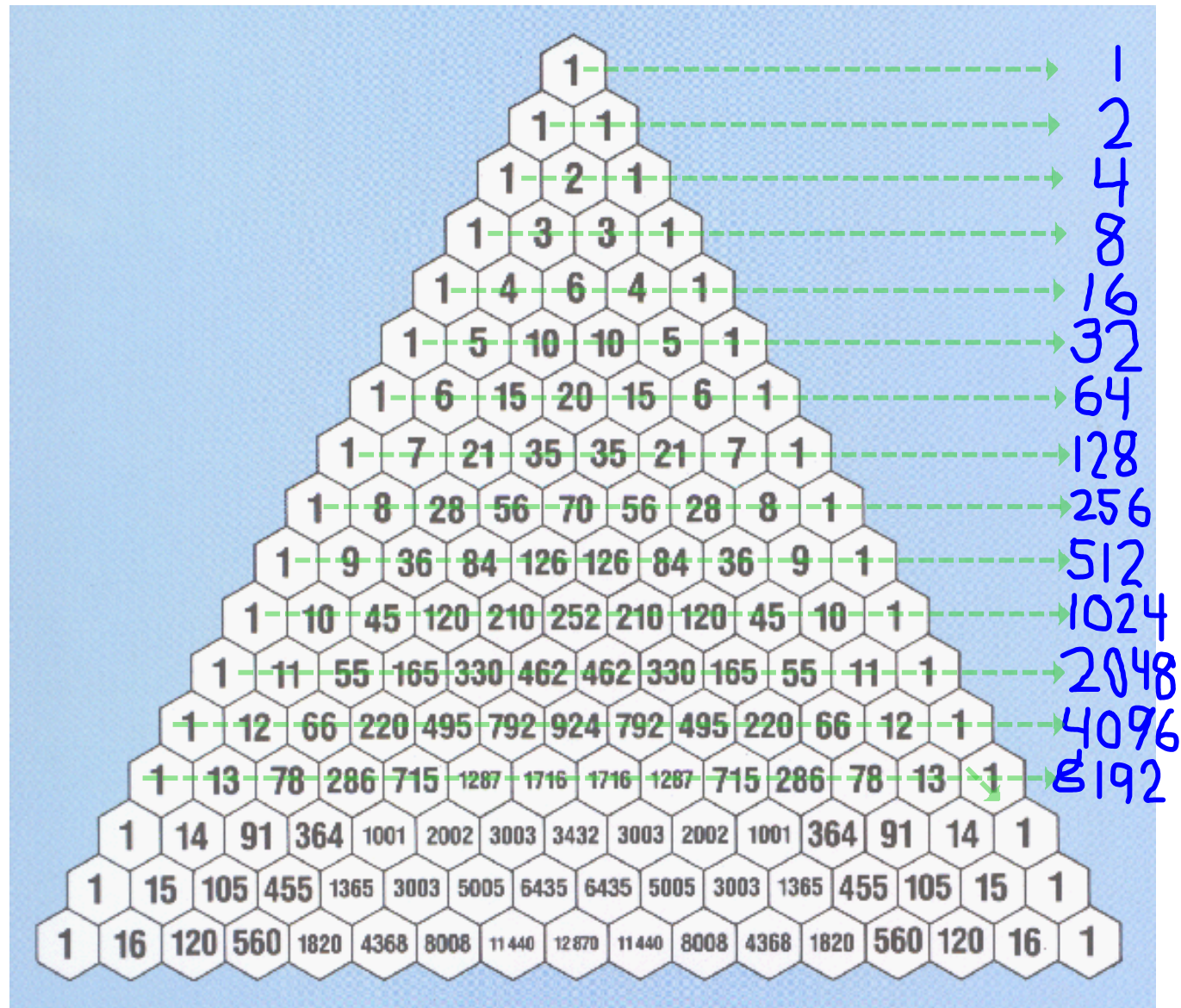
Step 1: Fill in the missing numbers in Pascal's Triangle

At the tip of Pascal's Triangle is the number 1, which makes up the zeroth row. The first row (1 & 1) contains two 1's, both formed by adding the two numbers above them to the left and the right, in this case 1 and 0 (all numbers outside the Triangle are 0's). Do the same to create the 2nd row: $0+1=1$; $1+1=2$; $1+0=1$. And the third: $0+1=1$; $1+2=3$; $2+1=3$; $1+0=1$. In this way, the rows of the triangle go on infinitely.

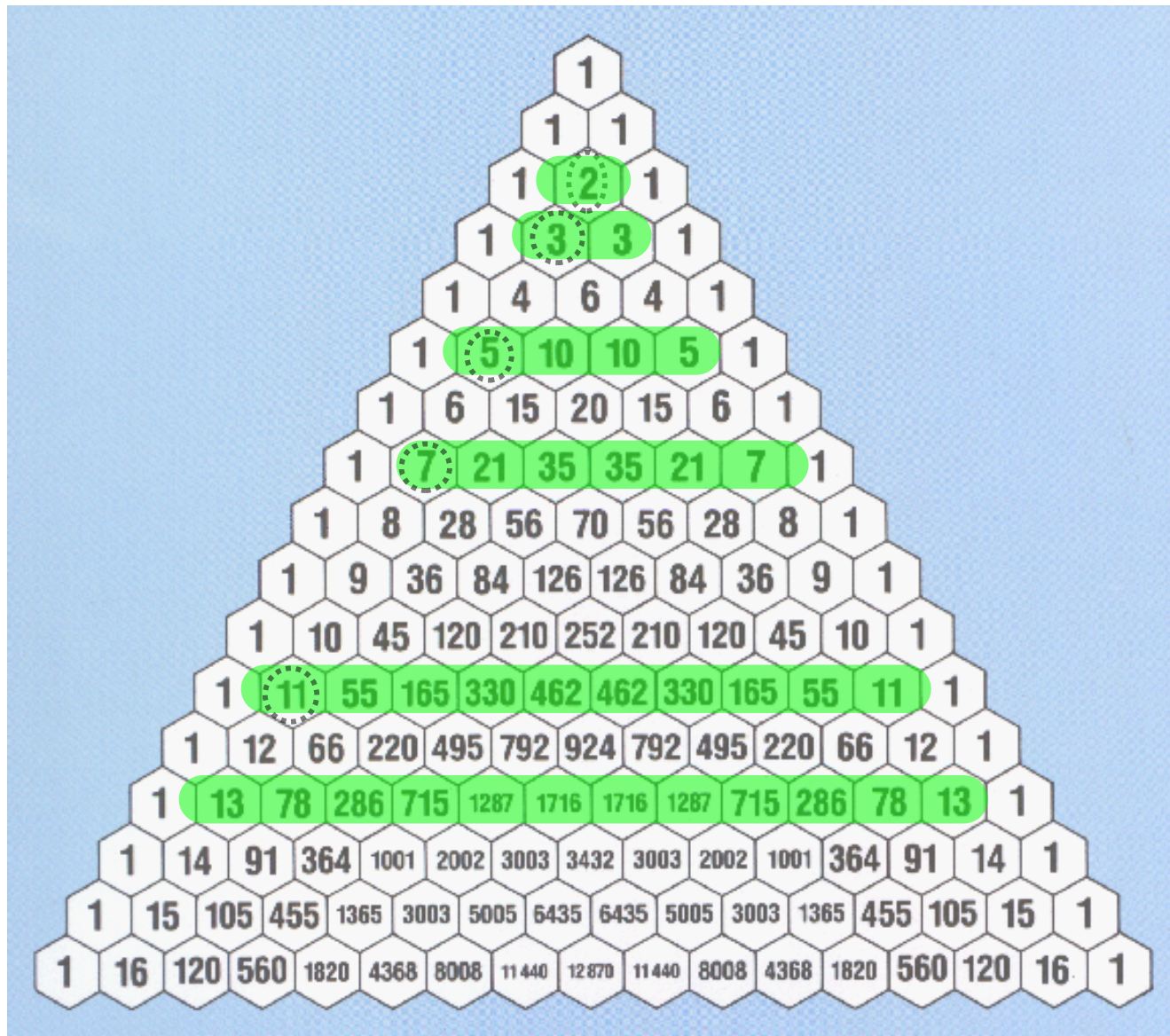




**THE SUM OF EACH CONSECUTIVE ROW FORMS THE
SERIES OF BINARY NUMBERS $(2^0 = 1 \text{ TO } 2^n)$**

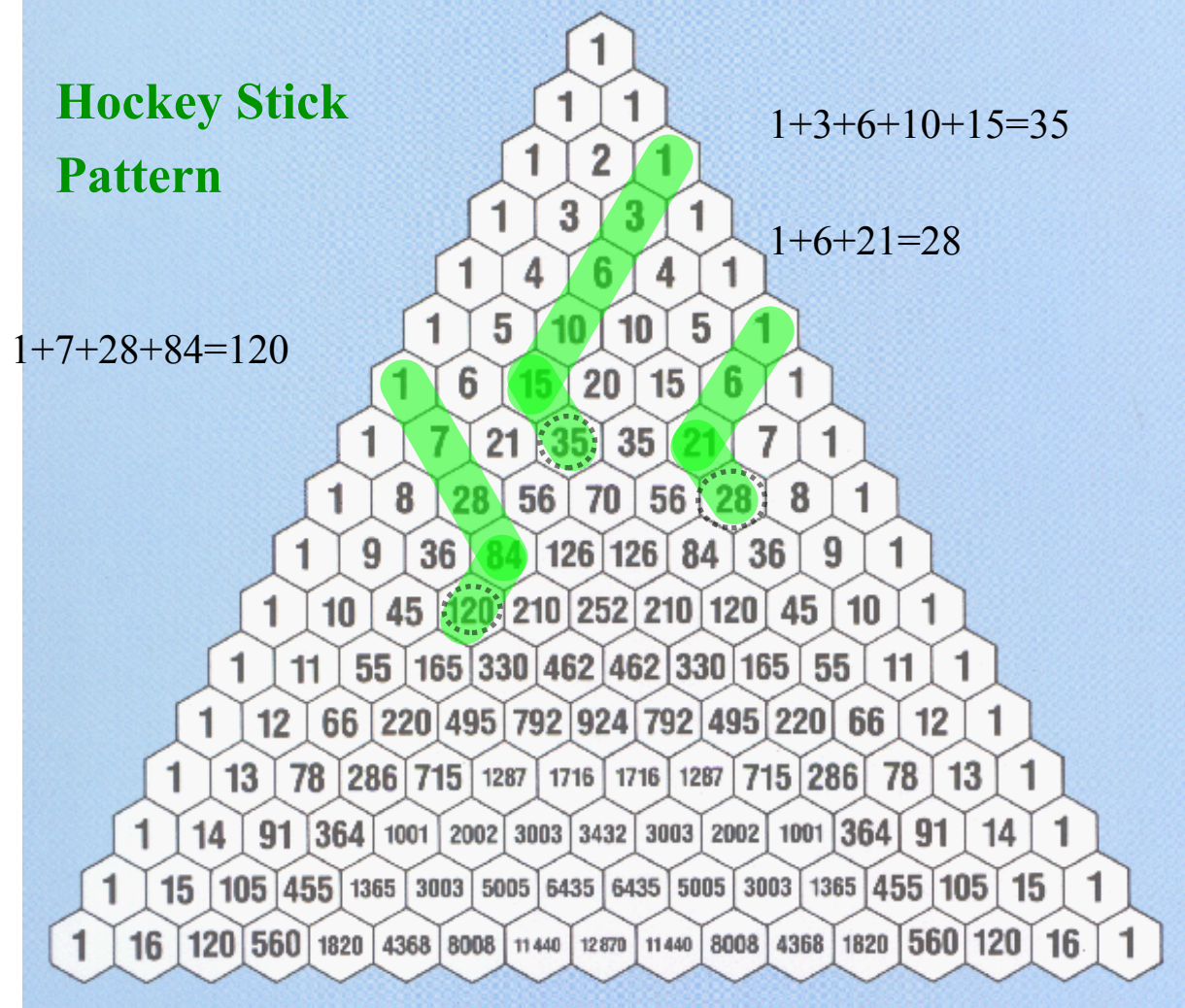


If the 1st element in a row is a prime number (remember, the 0th element of every row is 1), all the numbers in that row (excluding the 1's) are divisible by it. For example, in row 7 (1 7 21 35 35 21 7 1) 7, 21, and 35 are all divisible by 7.

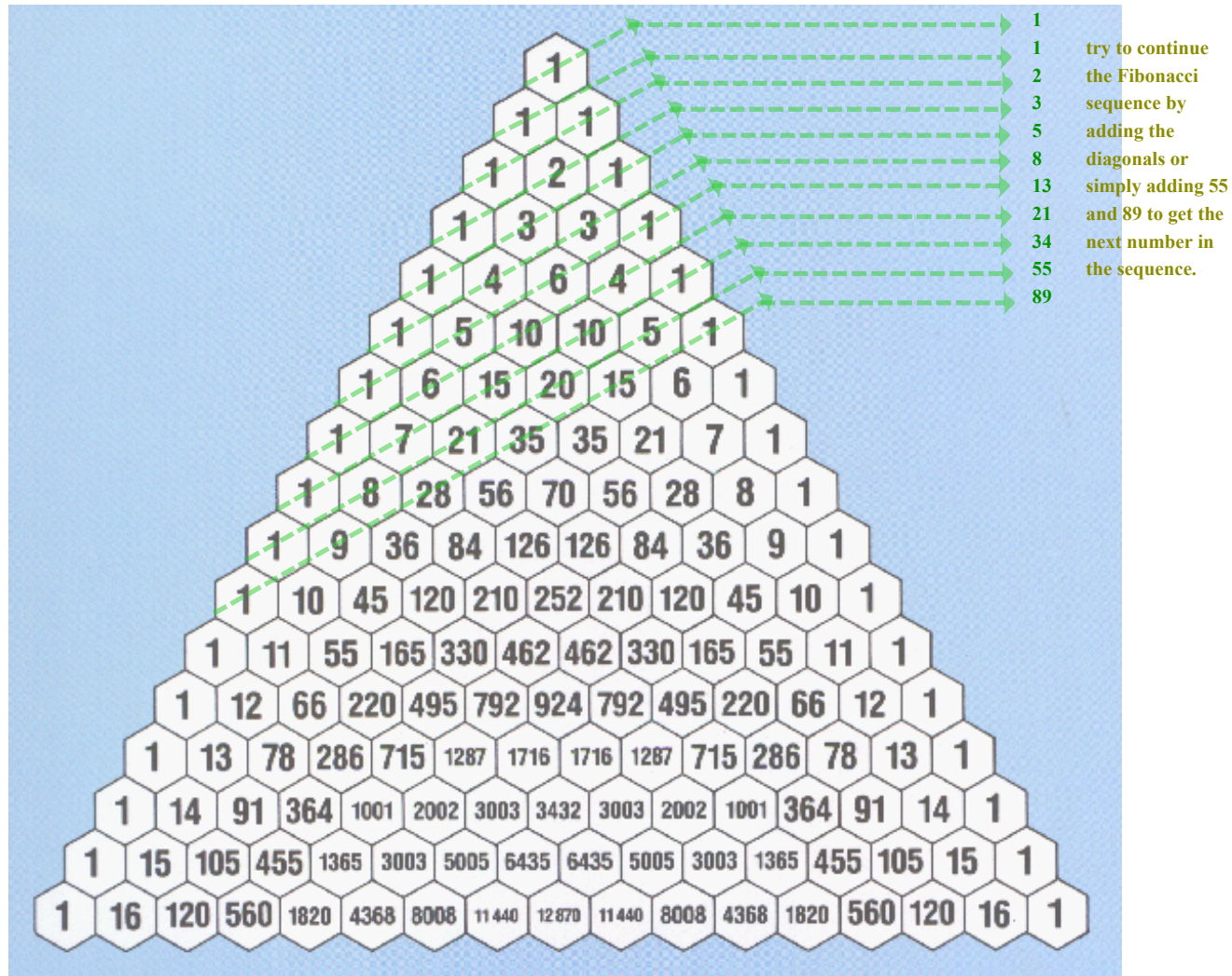


If a diagonal of numbers of any length is selected starting at any of the 1's bordering the sides of the triangle and ending on any number inside the triangle on that diagonal, the sum of the numbers inside the selection is equal to the number below the end of the selection that is not on the same diagonal itself.

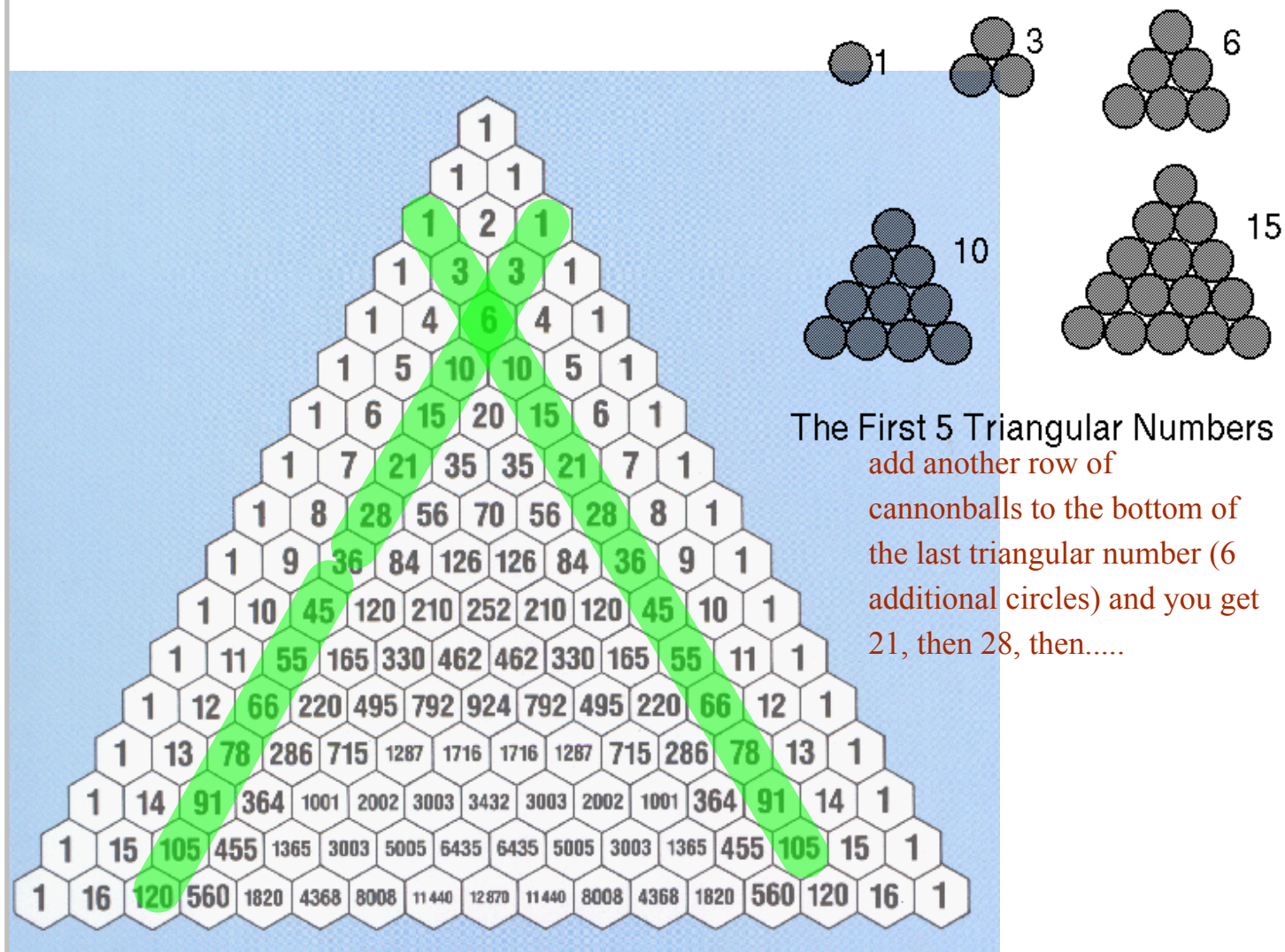
Hockey Stick Pattern



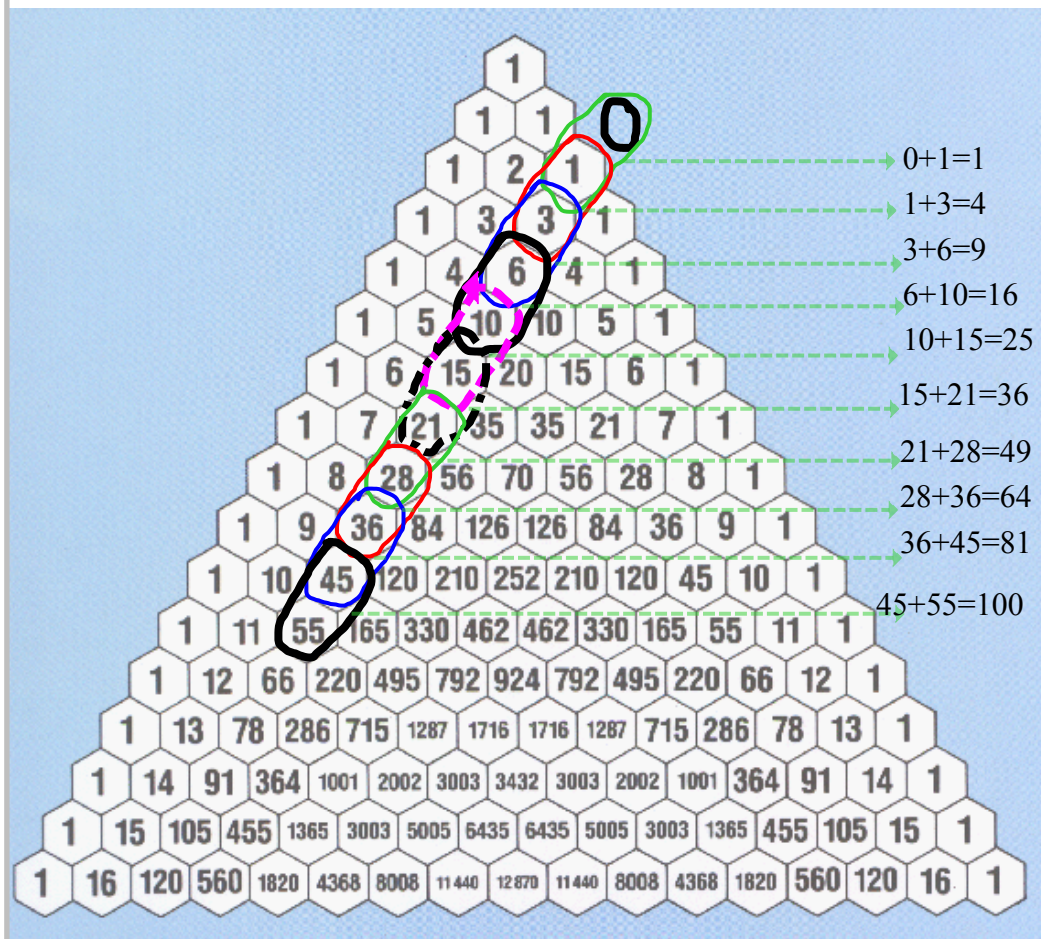
Fibonacci's Sequence can also be located in Pascal's Triangle. The sum of the numbers in the consecutive rows shown in the diagram are the first numbers of the Fibonacci Sequence. The Sequence can also be formed in a more direct way, very similar to the method used to form the Triangle, by adding two consecutive numbers in the sequence to produce the next number. The creates the sequence: 1,1,2,3,5,8,13,21,34, 55,89,144,233, etc



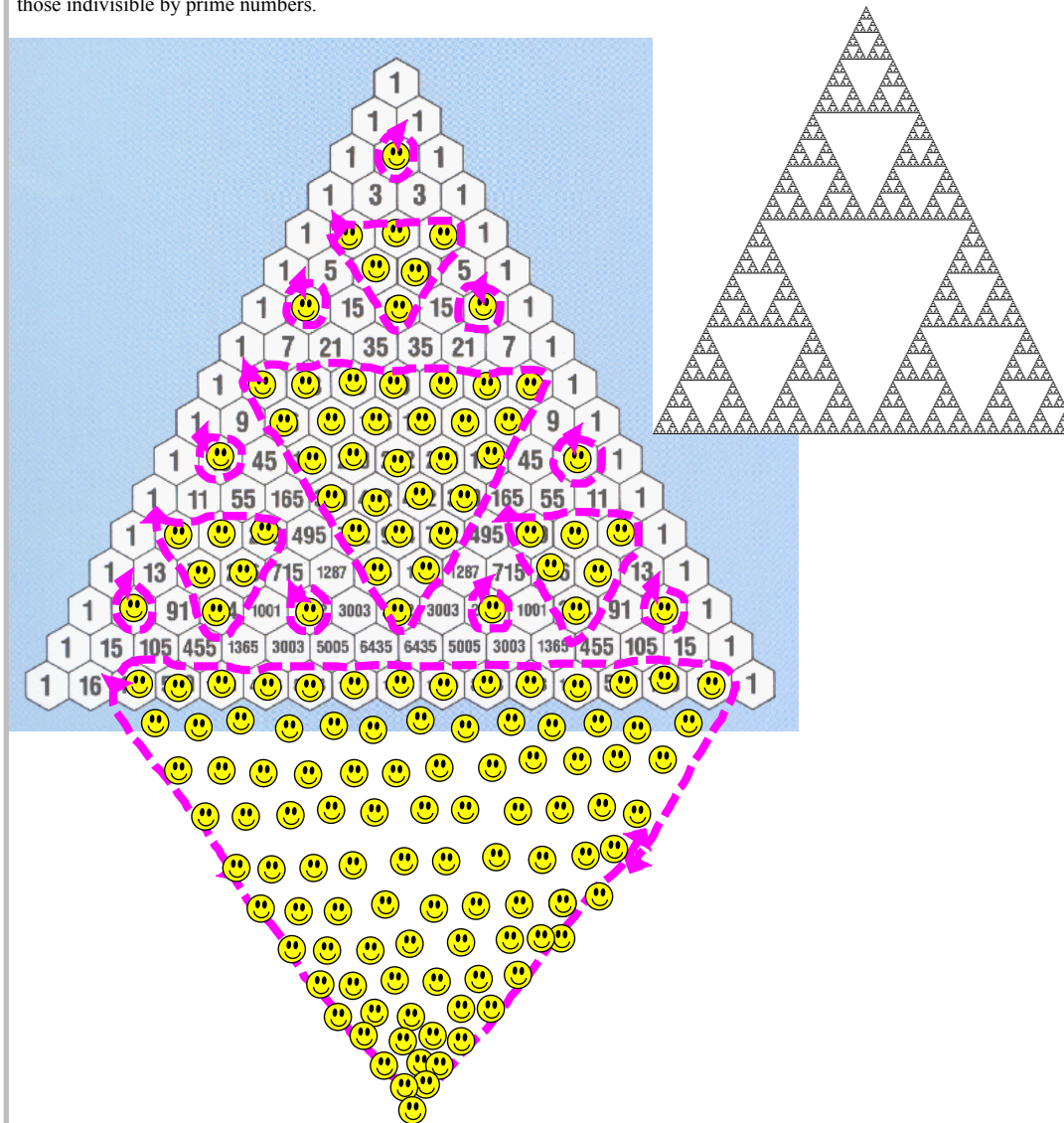
Triangular Numbers are just one type of polygonal numbers. The triangular numbers can be found in the diagonal starting at row 3 as shown in the diagram. The first triangular number is 1, the second is 3, the third is 6, the fourth is 10, and so on.



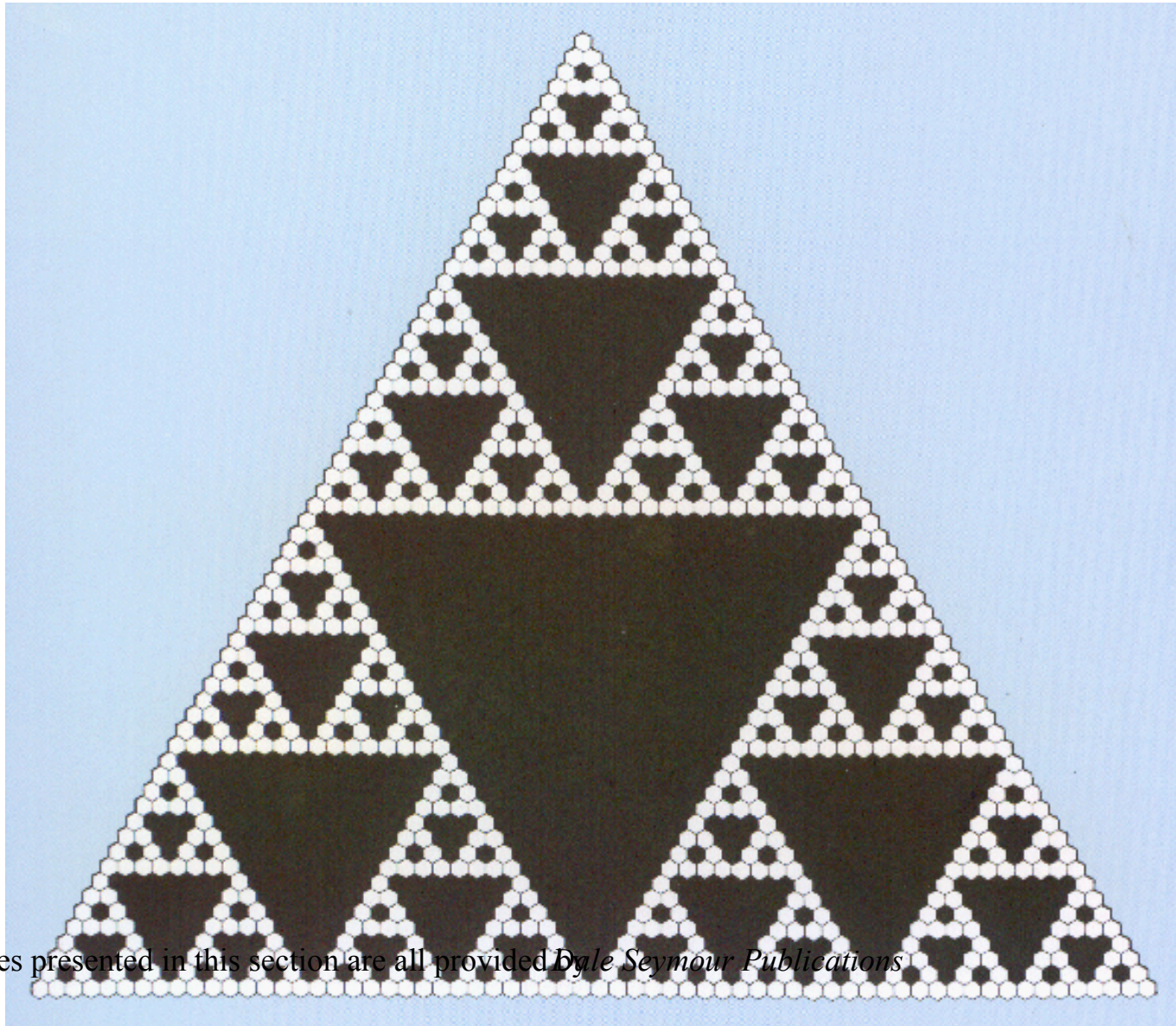
Square Numbers are another type of **Polygonal Numbers**. They are found in the same diagonal as the triangular numbers. A Square Number is the sum of the two numbers in any circled area in the diagram. (The colors are different only to distinguish between the separate "rubber bands").



When all the odd numbers (numbers not divisible by 2) in Pascal's Triangle are filled in (black) and the rest (the evens) are left blank (white), the recursive Sierpinski Triangle fractal is revealed (see figure at near right), showing yet another pattern in Pascal's Triangle. Other interesting patterns are formed if the elements not divisible by other numbers are filled, especially those indivisible by prime numbers.

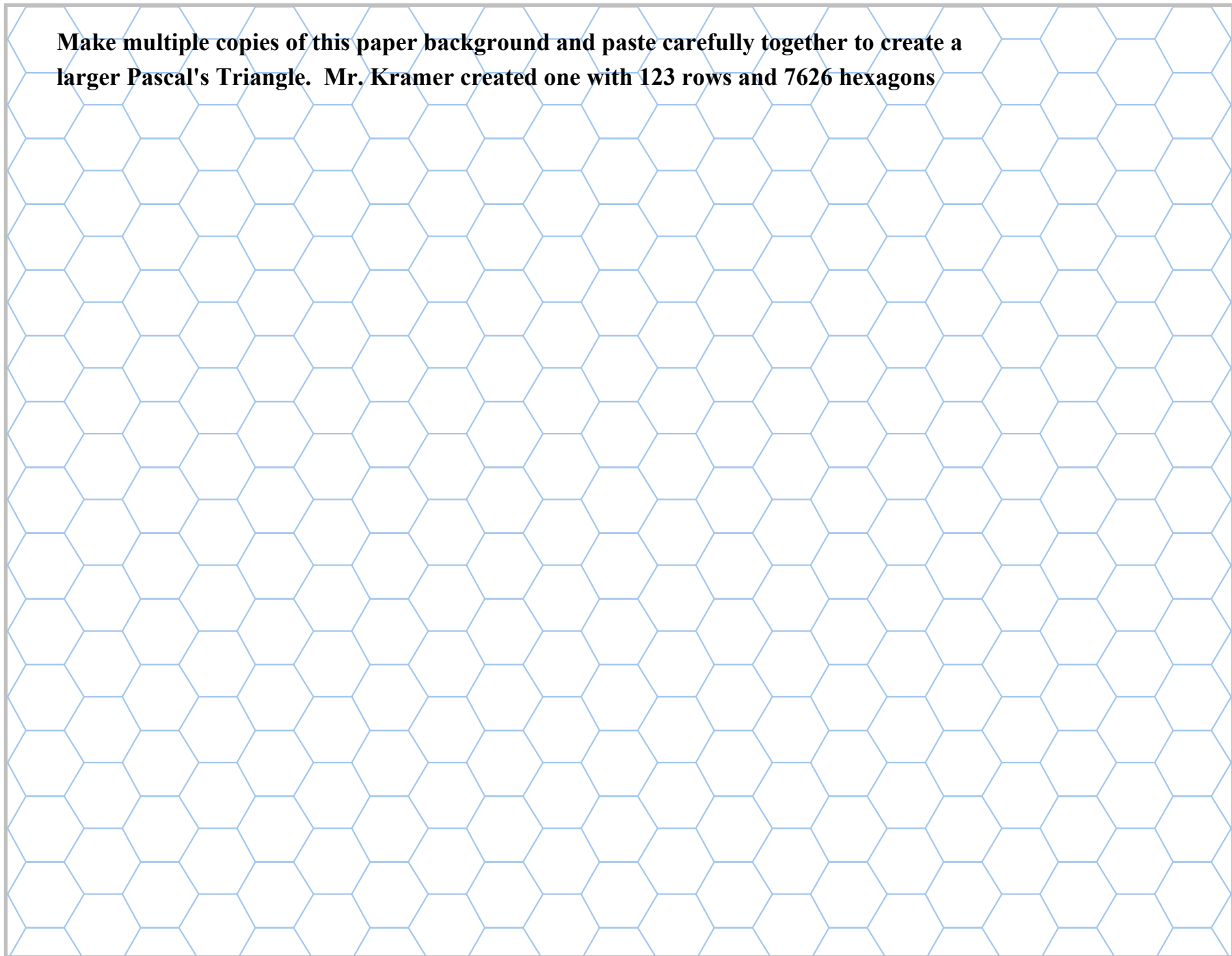


Step 2: Color the odd and even numbers with two distinct colors



Pictures presented in this section are all provided *Dyle Seymour Publications*

Make multiple copies of this paper background and paste carefully together to create a larger Pascal's Triangle. Mr. Kramer created one with 123 rows and 7626 hexagons





Title: Hexangular grid - medium (13 of 14)

For those of you that understand factorials and algebra, may attempt the following:

A number in the triangle can also be found by nCr (n Choose r) where n is the number of the row and r is the element in that row. For example, in row 3, 1 is the zeroth element, 3 is element number 1, the next three is the 2nd element, and the last 1 is the 3rd element. The formula for nCr is:

$$\frac{n!}{r!(n-r)!}$$

